Es 1 2022-23 1.1

1.A S1: prec. statica

S2: $d = \text{cost} \rightarrow A \geq 99\%$

S1 → gratis

S2 → $R(s) = K_R \cdot R'(s)$ $R'(0) = 1$ NO POLI ORIGINE

? vincolo su K_R per S2

formule $y_\infty = \dfrac{\cancel{D}}{K_R} \leq 0.01 \cancel{D} \rightarrow \underline{\underline{K_R \geq 100}}$

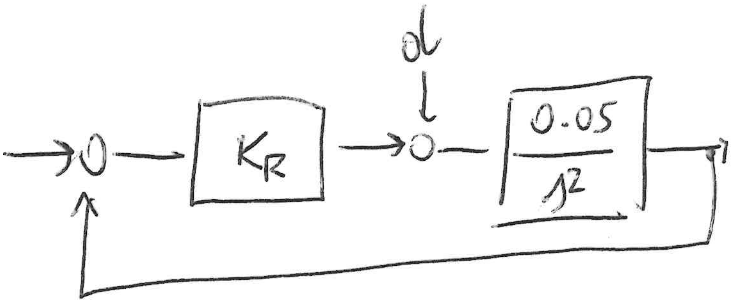
Deduciamo x altra via, imponendo che $\boxed{W_d^y(0) \leq 0.01}$

$$d = D \longrightarrow \boxed{W_d^y(s)} \xrightarrow{y} \quad y \rightarrow D \cdot W_d^y(0) \quad \text{TFRG}$$

$W_d^y(0)$ può calcolarsi in maniera + facile se poniamo $R(s) = K_R$

se infatti consideriamo $R(s)$ completo, nel momento in cui $s = 0$

$R(s) \rightarrow R(0) = K_R$



$$W_d^y = \frac{P}{1+RP} = \frac{\frac{0.05}{s^2}}{1 + K_R \frac{0.05}{s^2}} = \frac{0.05}{s^2 + K_R\, 0.05}$$

$$W_d^y(0) = \frac{1}{K_R}$$

conti "rigorosi"

$$W_d^y = \frac{\frac{0.05}{s^2}}{1 + K_R \cdot R'(s)\, \frac{0.05}{s^2}} = \frac{0.5}{s^2 + K_R\, R'(s)\, 0.05} \xrightarrow{s\to 0} \frac{1}{K_R}$$

$$\frac{1}{K_R} \le 0.01 \longrightarrow K_R \ge 100$$

Regolatore con $\nexists$ $K_R \ge 100$ che stabilizza rende il sistema a c.c. asintoticamente stabile.

1.3

Proviamo $R(s) = K_R$

Pcar : $F = RP = \frac{5}{s^2}$

Sommiamo NUM e DEN : $P_{CAR} = s^2 + 5$ poli immaginari puri NO

Proviamo con coppia P-Z

$$R(s) = K_R\left(\frac{s+z}{s+p}\cdot\cancel{\frac{p}{z}}\right)$$

$$F = RP = K_R\cancel{\frac{p}{z}}\,\frac{s+z}{s+p}\cdot\frac{0.05}{s^2}$$

$$P_{CAR}: \cancel{z}\, s^2(s+p) + 0.05\,K_R\,\cancel{p}\,(s+z)$$

$$= \cancel{z}\, s^3 + zp\, s^2 + 0.05\,K_R\,\cancel{p}\, s + 0.05\,K_R\, z\, p$$

$$\cancel{0.05\,K_R}\, z\, p^2 > \cancel{0.05\,K_R}\, z\, p$$

$$\boxed{p > z}$$

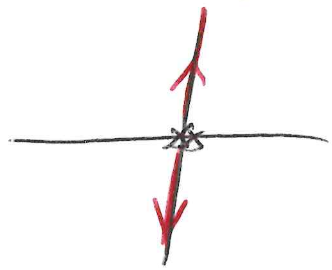
1.4

$p = 2 \quad z = 1$

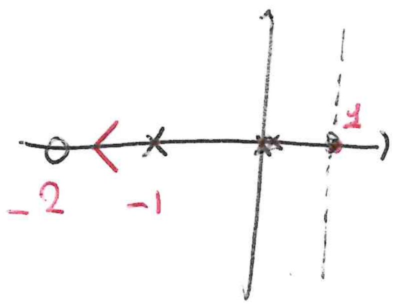
$$R(s) = 100 \, \frac{s+1}{s+2} = 200\left(\frac{s+1}{s+2}\right)$$

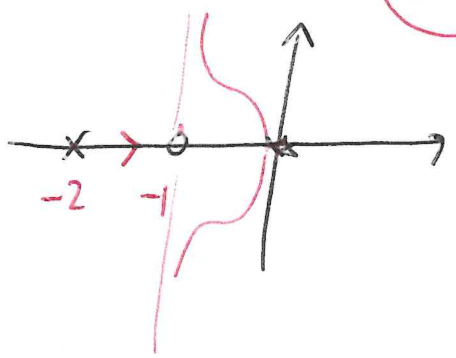
Ragionamento su LdR

$R(s) = K_R \qquad L(s) = \dfrac{0.05}{s^2}$



$$x_s = \frac{\sum p_i - \sum z_i}{n-m} = 0$$





$p > z$

Mettere coppia pz con il polo + a sinistra

es.

$p = 4 \qquad z = 3$

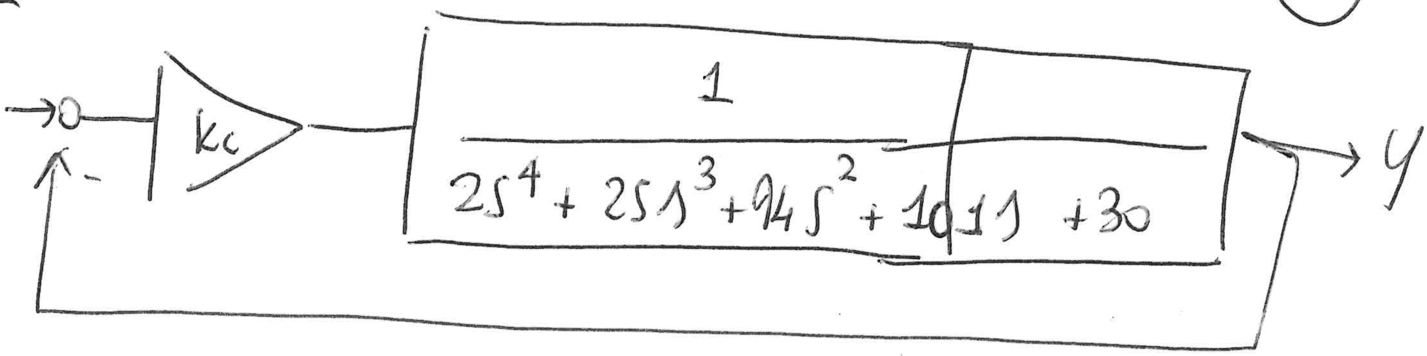
$$R = 100 \cdot \frac{4}{3} \, \frac{s+3}{s+4} = \frac{400s + 1200}{3s + 12}$$

1.B

$$3\dot{u} + 12u = 400\dot{e} + 1200e$$

EJ2

2.1



Pcar: $2s^4 + 25s^3 + 94s^2 + 101s + 30 + K_c$

2	94	$30+K_c$
25	101	
A	B	
C		

$$A = -\begin{vmatrix} 2 & 94 \\ 25 & 101 \end{vmatrix} = 2148$$

$$B = -\begin{vmatrix} 2 & 30+K_c \\ 25 & 0 \end{vmatrix} = 750 + 25K_c$$

2	94	$30+K_c$
25	101	
2148	$750+25K_c$	
C		

2.a

$$C = -\begin{vmatrix} 25 & 101 \\ 2148 & 750+25K_c \end{vmatrix} =$$

$$= 2148 \times 101 - 25(750 + 25K_c)$$

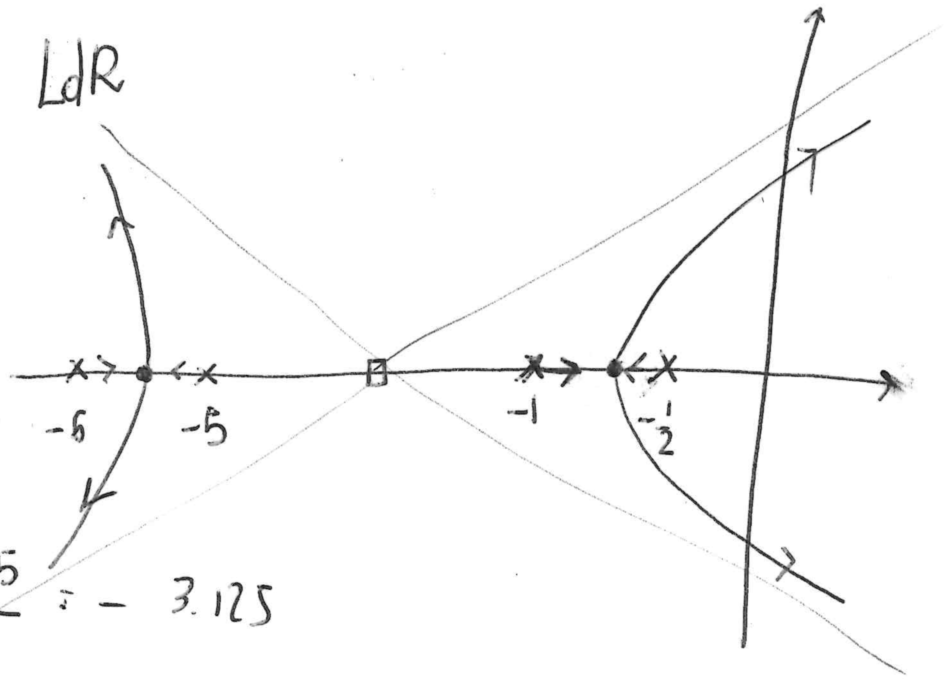
$$= 216948 - 18750 - 625K_c$$

$$= 198198 - 625K_c$$

$C > 0$ si $\quad K_c < \dfrac{198198}{625} = 317.11$

$K_{CR} = 317.11$

2.B LdR



$$x_s = -\frac{12.5}{4} = -3.125$$

2.3

Eq. punti doppi

$$\sum \frac{1}{s-p_i} = \underbrace{\cancel{\sum \frac{1}{s-z_i}}}_{0} =$$

$$\frac{1}{s+0.5} + \frac{1}{s+1} + \frac{1}{s+5} + \frac{1}{s+6} = 0$$

$$\frac{\text{POLIN DI } 3^\circ \text{ GRADO}}{(\quad)(\quad)(\quad)(\quad)} = 0$$

I punti doppi non possono essere valutati a mano.

$k^* =$ valore di guadagno <u>più piccolo</u> fra quelli associati ai 2 punti doppi

$$W_r^y = \frac{\dfrac{K_c}{2s^4 + \dots + 30}}{1 + \dfrac{K_c}{2s^4 + \dots + 30}} = \frac{K_c}{2s^4 + 25s^3 + 94s^2 + 101s + 30 + K_c}$$

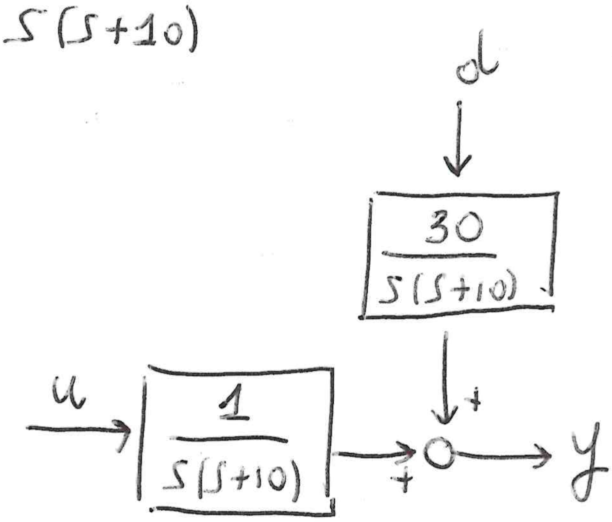
$$W_r^y(0) = \frac{K_c}{30 + K_c}$$ = valore di regime della y quando $r = 1$

(2.4)

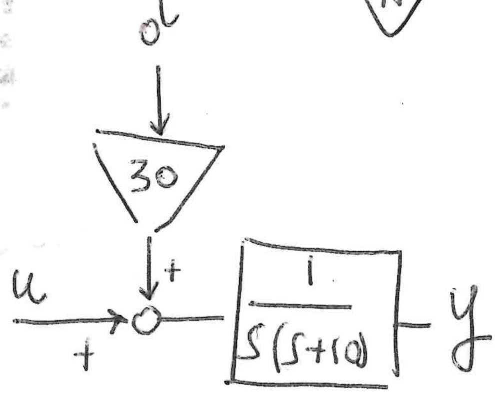
Es.3

$$F_u^y(s) = \frac{1}{s^2 + 10s} = \frac{1}{s(s+10)}$$

$$F_d^y(s) = \frac{30}{s(s+10)}$$



$\Downarrow$



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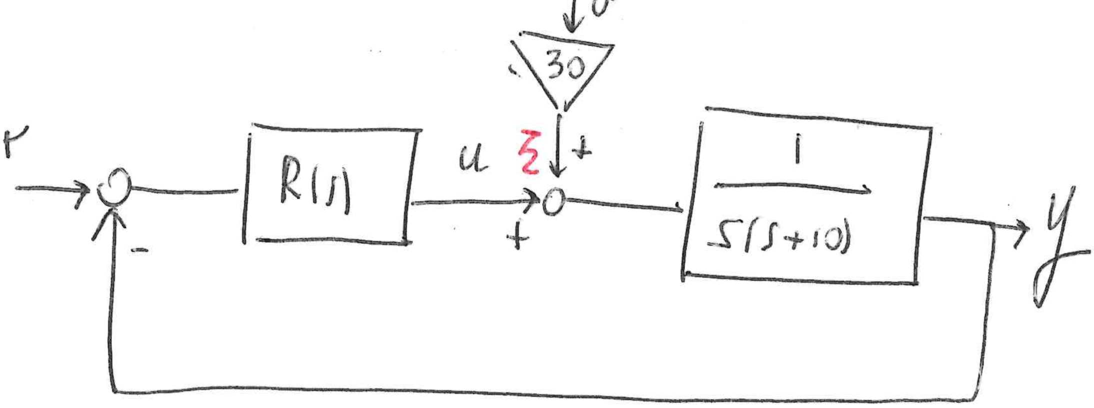


schema 1

3.2

S1 + S2 non serve polo nell'origine nel regolatore

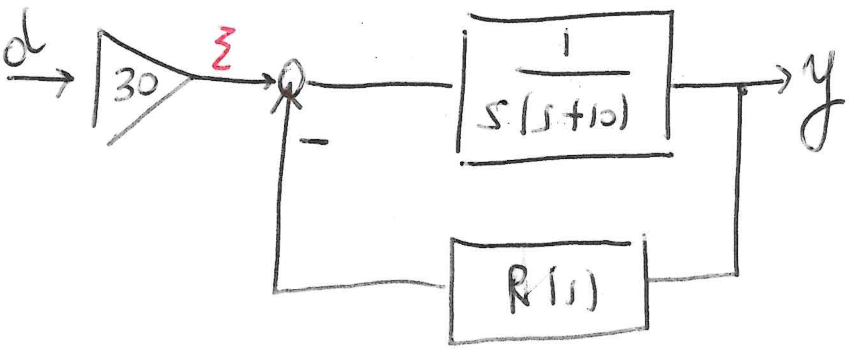
$R(s) = K_R R'(s) \qquad R'(0) = 1 \qquad (\nu = 0)$

S1 → nessun vincolo su K_R

S2 → vincolo, determiniamolo

S2 → $W_d^y(0) < 0.05$

Per la determinazione di $W_d^y(s)$ è conveniente riferirsi alla seguente rappresentazione



schema 2

$$W_\xi^y = \frac{\frac{1}{s(s+10)}}{1 + K_R R'(s) \cdot \frac{1}{s(s+10)}} = \frac{1}{s(s+10) + K_R R'(s)}$$

↓ poteva essere dedotto anche dallo schema 1

3.3

$$W_d^y = 30 \cdot W_z^y = \frac{30}{s(s+10) + K_R R'(s)}$$

$$W_d^y(0) = \frac{30}{K_R} < 0.05 \rightarrow K_R \geq 600$$

Quindi: $R(s) = K_R R'(s)$ con $\boxed{\begin{array}{l} R'(0) = 1 \\ K_R \geq 600 \end{array}}$

potrebbe essere sufficiente $R'(s) = 1$, cioè un reg proporzionale.

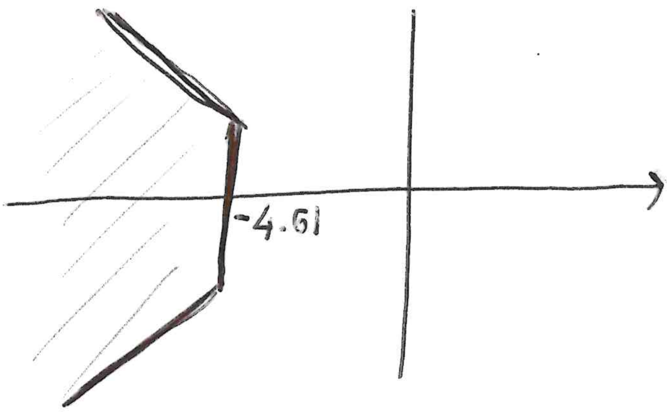
Deve essere garantita la stab a ciclo chiuso e l'appartenenza dei poli alla reg. ammissibile.

Calcoliamo la reg. Ammissibile:

(3.4)

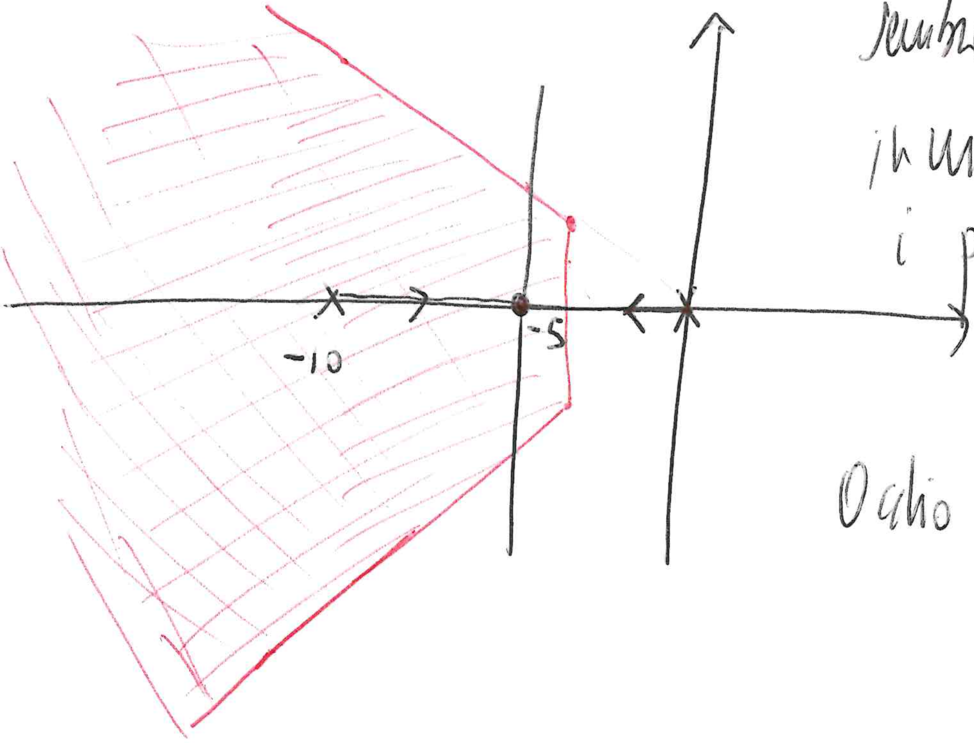
(S3) $A^* = -\frac{6}{T^*} = -\frac{6}{1.3} = -4.61$

(S4) $\xi \geq 0.7$



Vediamo se con un $R(s) = K_R$ ce la si fa

$L(s) = \frac{1}{s(s+10)}$



sembra possibile fare in modo che con $R = K_R$ i poli ricadano nella regione ammissibile.

Occhio al $K_R \geq 600$.

3.5

vediamo dove stanno i poli con $K_R = 600$

P_{CAR}: $s(s+10) + K_R = s^2 + 10s + 600$

$\Delta < 0 \rightarrow$ poli complessi coniugati. ($Re = -5$)

Smorzamento ?

$$s^2 + 10s + 600 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{600} = 24.49$$

$2\zeta\omega_n = 10$ $\qquad$ $\zeta = \frac{10}{2\omega_n} = 0.2$ $\boxed{NO}$

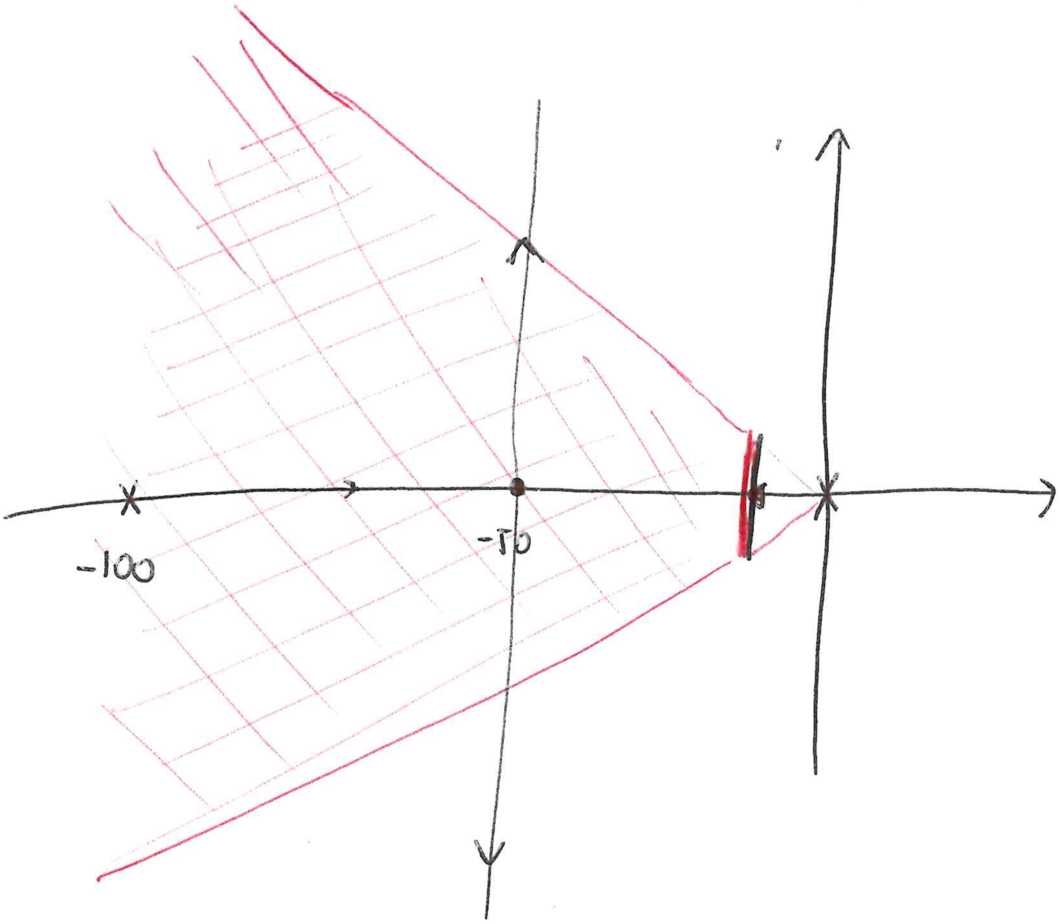
Serve un $R'(s)$.

Cancelliamo il polo in -10 e "spostiamolo più a sinistra".

3.6

$$R(s) = K_R \cdot 10 \cdot \frac{s+10}{s+100}$$

$$L(s) = 10 \, \frac{\cancel{s+10}}{s+100} \cdot \frac{1}{s\,(\cancel{s+10})} = \frac{10}{s\,(s+100)}$$



Vediamo dove stanno i poli quando $K_R = 600$

P_{CAR}: $s^2 + 100\,s + 10\,K_R = s^2 + 100\,s + 6000$

$\Delta = 10000 - 24000 < 0$ $\quad (Re = -50)$

(3.7)

$$s^2 + 100s + 6000 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n^2 = 6000 \rightarrow \omega_n = \sqrt{6000} = 77.45$$

$$2\zeta\omega_n = 100 \qquad \zeta = \frac{100}{2\omega_n} = \frac{100}{154.91} = 0.64 \ \boxed{NO}$$

Vediamo con

$$R(s) = K_R \, 15 \, \frac{s+10}{s+150}$$

$$L(s) = 15 \, \frac{\cancel{s+10}}{s+150} \, \frac{1}{s(\cancel{s+10})} = \frac{15}{s(s+150)}$$

$$P_{CAR}: \ s^2 + 150s + 15K_R = s^2 + 150s + 9000$$

$$\omega_n = \sqrt{9000} = 94.86$$

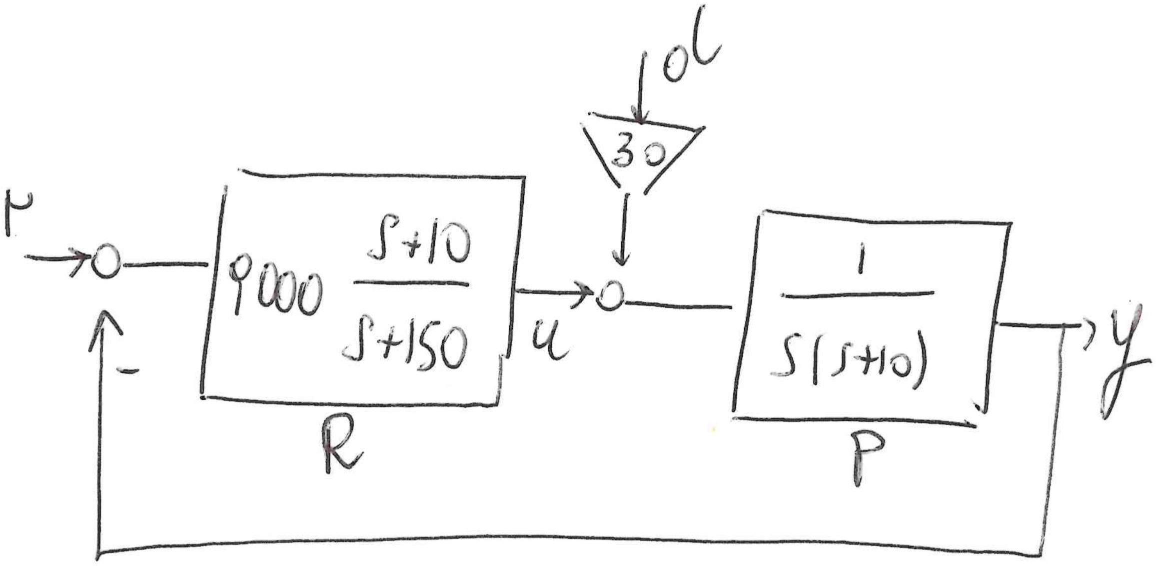
$$2\zeta\omega_n = 150 \rightarrow \zeta = \frac{150}{2\omega_n} = 0.79 \ \boxed{OK}$$

$$R(s) = 600 \cdot 15 \cdot \frac{s+10}{s+150} = 9000 \, \frac{s+10}{s+150}$$

3.b

3.8

$$R(s) = 9000 \, \frac{s+10}{s+150}$$



$r = 5 \longrightarrow y \to 5$ PMI

$r = 2t$ PMI non può essere ottenuto

calcoliamo il valore di regime dell'<u>errore</u> (se $\exists$ finito)

per sistemi di contr. di tipo 1, l'errore tende a:

$$e(t) \to E_\infty = \frac{\Sigma}{\mu_R \, \mu_P}$$

Σ = pendenza rampa

μ_R = guadagno statico (eventualmente generalizzato) del controllore

μ_P = " " del processo

3.9

$\Sigma = 2$

$\mu_R = 600 \qquad \mu_P = 0.1$

$$E_\infty = \frac{2}{600 \times 0.1} = \frac{2}{60} = 0.03$$

$e(t) = r(t) - y(t) \rightarrow 0.03$

$y_{regime} = r(t) - 0.03 = 2t - 0.03$

calcoliamo lo 0.03 per altra via.

$$W_r^e = \frac{1}{1 + RP} = \frac{1}{1 + \frac{9000}{s(s+150)}} = \frac{s(s+150)}{s(s+150) + 9000}$$

$$RP = 9000 \frac{\cancel{s+10}}{s+150} \cdot \frac{1}{s\cancel{(s+10)}} = \frac{9000}{s(s+150)}$$

3.10

$$\frac{s(s+150)}{s(s+150)+9000}$$

$$r(t) \rightarrow \boxed{} \rightarrow e$$

$$E(s) = \frac{s(s+150)}{s(s+150)+9000} \cdot \frac{2}{s^2} = \frac{2(s+150)}{s[s(s+150)+9000]}$$

polo semplice in $s=0$

2 poli a $Re<0$

ok, CN soddisfatta

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{2(s+150)}{s(s+150)+9000} = \frac{2\cdot 150}{9000} = 0.033$$

$$y_{regime} = 2t - 0.033$$

3.11) $P(t) = 5 \sin(20t + 0.1)$

$$y_{regime}(t) = 5 \cdot M \cdot \sin(20t + 0.1 + \phi)$$

$M_{db} \simeq -40\ db$ $\qquad \phi^{deg} = -15^\circ$

$M = 10^{\frac{-40}{20}} = 0.01$ $\qquad \phi^{rad} = -15 \cdot \frac{2\pi}{360} = -0.26$

$$y_{reg} = 0.05 \sin(20t + 0.1 - 0.26)$$

$d = 0.2 = D$

$$y \longrightarrow D \cdot W_d^y(0) = 0.2 \times \frac{30}{K_R} =$$

$$= 0.2 \times \frac{30}{600} = 0.01$$

3.12

unendo tutto

$$y_{regime}(t) = 5 + (2t - 0.03) +$$

$$+ 0.05 \sin(20t + 0.1 - 0.26) + 0.01$$

4.1

~~$X(s) =$~~

$X(s)$ ha 2 poli sull'asse immaginario.

non soddisfa la cond. applicabilità

$$\frac{3s^2+3s+10}{(s+1)(s^2+4)} = \frac{2}{s+1} + \frac{s+2}{s^2+4}$$